Int J Thermophys (2009) 30:1025–1039
DOI 10.1007/s10765-009-0598-y

Determination of Thermal Properties in the Frequency Domain Based on a Non-integer Model: Application to a Sample of Concrete

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Published online: 11 June 2009

Abstract A technique for determining thermophysical properties is proposed and applied to a sample of concrete by taking advantage of pseudo-random signals. Data are treated in the frequency domain. A new approach is developed for estimating the thermal impedance based on the formalism of non-integer order models. An experimental setup consisting of a heat flux and temperature sensor arranged in contact with a material assuming a semi-infinite boundary condition is studied. The theoretical expression for such a thermal impedance takes into account the thermal capacity of the sensor and the contact resistance and emphasizes fractional orders in the behavior model.

Keywords Concrete · Contact resistance · Frequency domain · Non-integer order models · Thermal impedance · Thermophysical parameters

List of Symbols

a	Thermal diffusivity ($m^2 \cdot s^{-1}$)
b	Thermal effusivity ($J \cdot m^{-2} \cdot s^{-1/2} \cdot K^{-1}$)
$\bar{b}$	Average value of the thermal effusivity
c	Specific heat capacity ($J \cdot kg^{-1} \cdot K^{-1}$)
C	Thermal capacity of the system ($J \cdot m^{-2} \cdot K^{-1}$)
C_f	Fluxmeter capacity ($J \cdot m^{-2} \cdot K^{-1}$)

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C_{v}	Coefficient of variation
D	Differentiation operator
f	Frequency (Hz)
h	Sampling period (s)
j	Complex variable
ℓ	Thickness of material (M)
$n_{\alpha_I}, n_{\beta_J}$	Derivative orders
p	Laplace variable
p_i	Theoretical impedance parameter
S_{p_i}	Impedance sensitivity function to parameter p_i
R	Thermal resistance of the system ($\mathrm{K \cdot m^2 \cdot W^{-1}}$)
R_{c}	Contact resistance ($\mathrm{K \cdot m^2 \cdot W^{-1}}$)
R_{f}	Fluxmeter resistance ($\mathrm{K \cdot m^2 \cdot W^{-1}}$)
t	Time (s)
T	Temperature (K)
Z_{e}	Thermal input impedance ($\mathrm{K \cdot m^2 \cdot W^{-1}}$)
Z_{c}	Characteristic thermal impedance ($\mathrm{K \cdot m^2 \cdot W^{-1}}$)
Z_{th}	Theoretical Impedance ($\mathrm{K \cdot m^2 \cdot W^{-1}}$)
λ	Thermal conductivity ($\mathrm{W \cdot m^{-1} \cdot K^{-1}}$)
θ	Temperature (°C)
ρ	Density ($\mathrm{kg \cdot m^{-3}}$)
ρc	Volumetric heat capacity ($\mathrm{J \cdot m^{-3} \cdot K^{-1}}$)
ϕ	Heat flux ($\mathrm{W \cdot m^{-2}}$)
σ	Standard deviation
α_i, β_j	Differential model parameters

1 Introduction

The determination of thermophysical properties (thermal conductivity, specific heat, thermal effusivity, or thermal diffusivity) of materials is essential in many fields such as energy engineering, chemistry engineering, material engineering, civil engineering, etc. Many scientific studies deal with thermophysical property measurements. One can distinguish the methods of thermophysical characterization based on studies using a steady state and those using a transient regime. As the first methods require long time periods, making them unsuitable for numerous applications, researchers have focused on transient methods.

In order to estimate thermophysical properties, various types of sensors have been designed and developed. Such sensors have been designed with various geometries, shaped as disks, rings, strips, planes, and single- or double-stems. However, the procedure for determining these thermophysical properties remains more or less the same. It involves comparing temperature changes in the sensor with a time-dependent model as it is subjected to the dissipation of a given heat load. Expressing the time-dependent model classically requires simultaneously solving both the heat equation and Fourier's law. The solution leads to a time-dependent expression for the sensor temperature when the boundary conditions are simple and well-defined. Especially, it is mandatory to model the heat flux due to heating by means of a simple analytical expression that can be solved in a straightforward manner. Such a requirement considerably limits the type of stress and leads in most cases to consider a power step. Stringent compliance with the stress signal is therefore an essential condition for such methods to be successful. In situ, boundary conditions cannot be easily controlled. Natural exchanges are added to the imposed conditions that disturb the measurement.

Parameters of the time-dependent model are the thermophysical data of the studied material, as well as characteristics of both the sensor and the sensor/material contact. The characteristics of the sensor do not depend on time and may be estimated or determined via preliminary tests [1]. On the other hand, the contact between the sensor and material is a significant issue as the contact resistance is not one of the parameters identified for each test. Considering single cases, the contact resistance may be neglected or can be reproduced satisfactorily, and its value can be fixed at a nominal value in the model. However, in most cases, its value changes from one test to another and the variations lead to errors in estimating the thermophysical parameters of the material. For materials with a rough surface, the contact resistance is significant with a strong influence on the heat exchanges. The sensor–material assembly is subjected to heat dissipation and responds with a change in the temperature of the sensor—the only data recorded. Such an approach only takes into account the response of the system to be characterized.

By definition, the thermal impedance is the ratio between the spectra of the temperature signal on a given surface and the heat flow crossing it. Both quantities can be measured simultaneously using a fluxmeter fitted with a thermocouple. If the thermal impedance is estimated for characterization purposes, both the imposed stress and temperature response are observed simultaneously, and their interaction is analyzed. Such an analysis is carried out in the frequency domain, enabling the system to be studied on the basis of random signals. The frequency approach facilitates an understanding of the phenomena, and along with the sensitivity study, the transition to the spectral domain enables us to target precisely in which frequency range the optimal work can be selected to identify the required parameters.

The thermophysical parameters can be identified with the theoretical model to adjust the experimentally determined impedance. In the Laplace domain, the theoretical behavior model shows that both the flux density and temperature are related in a differential equation in which the orders of differentiation are fractional. Then we can introduce a new technique to determine thermal impedance values, based on non-integer order model theory [2].

The thermal impedance is generally estimated by classical Fourier transform techniques or on the basis of a parametric model of the z-transform type [3]. The approach introduced in this study allows us to use directly the theoretical model to estimate the experimental impedance. The theoretical model is a function of the thermal effusivity of the material and also takes into account the thermal capacity of the sensor and the contact resistance that is identified for each test. In this study, the validity of the model is discussed via a harmonic study, and the method is applied to a sample of concrete. Such a case is interesting because the thermal properties of concrete vary significantly from one composition to another and also with time [4]. The contact resistance may have large values, and it must be taken into account so as to avoid larges errors.

2 Experimental Setup and Thermal Impedance Modeling

Thermal impedance [5] is defined in the Laplace domain as the ratio of the temperature and heat flux Laplace transforms. Thus, $Z(p)$ can be written as

$$\mathrm{Z}(p) = \frac{\theta(p)}{\varphi(p)} \tag{1}$$

An advantageous theoretical expression of the impedance, as will be shown in the next part, can be expressed using a non-integer model.

The experimental procedure for calculating the apparent thermal impedance of the input side of a material requires both temperature and flux sensors to be arranged on the sample. Then one may observe either the natural exchanges between the material and the environment, or as in our case, heat the sensor/material assembly with a thermal resistance.

To ensure exchanges as one-directional heat flow, the sensor consists of a sensitive measuring area surrounded by a ring with a similar but inert material acting as a guard ring. The sensor is a tangential-gradient fluxmeter [6], 0.5 mm in thickness, whose temperature is measured with an embedded T-thermocouple as shown in Figs. 1, 2.

Impedance values depend on a triple-layer system. The capacity of the sensor C_f ($\mathrm{J \cdot m^{-2} \cdot K^{-1}}$) at the input of the system can be considered as the first layer. The second layer is the contact resistance R_c ($\mathrm{K \cdot m^2 \cdot W^{-1}}$) at the sensor–material interface, and the third one is the material (in a semi-infinite boundary condition) characterized by its thermal effusivity b ($\mathrm{J \cdot K^{-1} \cdot m^{-2} \cdot s^{-1/2}}$).

The formalism of thermal quadrupoles can be considered with a transfer matrix associated with both the sensor and contact resistance [7].

$$\begin{bmatrix} 1 & 0 \\ pC_\mathrm{f} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & R_\mathrm{c} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Rc \\ pC_\mathrm{f} & 1 + pR_\mathrm{c}C_\mathrm{f} \end{bmatrix} \tag{2}$$

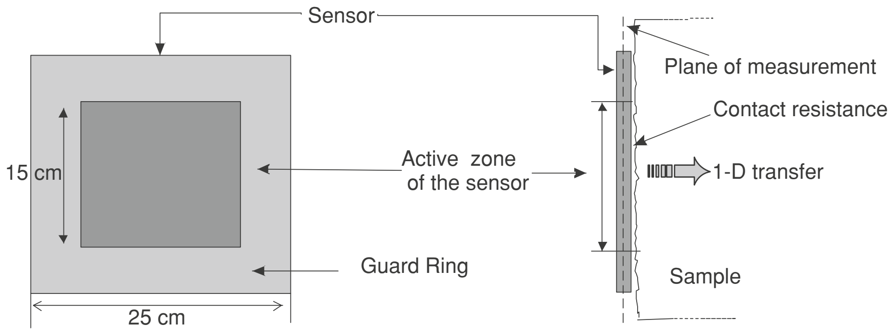

Fig. 1 Sensor

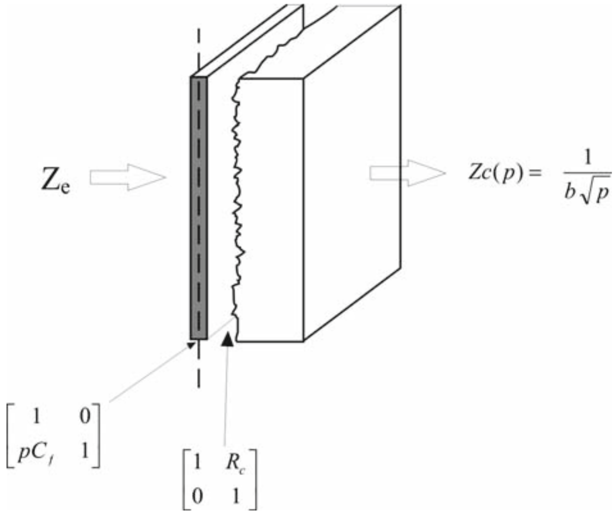

Fig. 2 Modeling parameters of the thermal system

As semi-infinite conditions can be assumed, the medium exhibits a characteristic impedance Z_c:

$$Z_c(p) = \frac{1}{b\sqrt{p}} \tag{3}$$

The parameter b (J · m^{-2} · s$^{-1/2}$ · K^{-1}) represents the thermal effusivity of the material. Such a parameter represents the material's ability to absorb heat from the medium in contact especially with transient exchanges. The product of the transfer matrices provides an expression for the impedance Z of the system in the Laplace domain;

$$Z(p) = \frac{1 + bR_c p^{1/2}}{bp^{1/2} + pC_f + bR_c C_f p^{3/2}} \tag{4}$$

Then the transfer function can be written as

$$Z\,(p) = \frac{\beta_1 + \beta_2 p^{1/2}}{\alpha_1 p^{1/2} + \alpha_2 p + \alpha_3 p^{3/2}} \tag{5}$$

The model highlights a linear relation between both the time-dependent input (heat flux $\varphi(t)$) and output (temperature $\theta(t)$) quantities by way of a differential equation whose orders of differentiation are fractional [2].

$$\alpha_1 \mathrm{D}^{1/2}\theta(t) + \alpha_2 \mathrm{D}\theta(t) + \alpha_3 \mathrm{D}^{3/2}\theta(t) = \beta_1\varphi(t) + \beta_2 \mathrm{D}^{1/2}\varphi(t) \tag{6}$$

Terms with D^n in this expression stand as generalized derivatives that can be calculated with the following recurrence equations:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^{n_\alpha}\theta(t)=\frac{1}{h^{n_\alpha}}\sum_{k=0}^{K}(-1)^k\binom{n_\alpha}{k}\theta\left(t-kh\right) \tag{7}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^{n_\alpha}\varphi(t)=\frac{1}{h^{n_\alpha}}\sum_{k=0}^{K}(-1)^k\binom{n_\alpha}{k}\varphi\left(t-kh\right) \tag{8}$$

with

$$\binom{n_\alpha}{k}=\frac{n_\alpha\left(n_\alpha-1\right)\cdots\left(n_\alpha-k+1\right)}{k!} \tag{9}$$

In this approach, h is the sampling period and n_α is the order of differentiation.

On the basis of Eq. 7, recordings of changes in flux density and surface temperature can be used to setup a system of equations in which coefficients α_i and β_j are the unknown parameters. By changing these variables [8], this system can be transcribed into a linear form. Coefficients α_i and β_j, which are combinations of the parameters of the experimental impedance, are obtained by matrix inversion.

The impedance in the frequency domain is a special case with transfer functions defined in the Laplace domain as $p=\mathrm{j}2\pi f$ ($\mathrm{j}=\sqrt{-1}$ and f is the frequency in Hz). The thermophysical parameters of the system (especially the thermal effusivity b) can be estimated by fitting the theoretical impedance model to the experimental impedance. An iterative procedure allows the fitting method to minimize the gap between both impedance values.

Z(f) is a complex function that can be expressed as

$$Z\left(f\right)=\frac{1+bR_\mathrm{c}\left(\mathrm{j}2\pi f\right)^{1/2}}{b\left(\mathrm{j}2\pi f\right)^{1/2}+\mathrm{j}2\pi fC_\mathrm{f}+bR_\mathrm{c}C_\mathrm{f}\left(\mathrm{j}2\pi f\right)^{3/2}} \tag{10}$$

As we will see, a short observation time means working in a domain where the sensor has a perturbing effect; then the measurement depends on three components of the thermal system: the sensor, the sensor–material contact resistance, and the material under study, according to its thermal effusivity.

3 Analysis of Impedance Sensitivity to Thermophysical Parameters

The objective of such a sensitivity study is to define the influence of each system parameter and to optimize the choice of the frequency range to be used for identification purposes.

In an inverse technique procedure, thermophysical parameters are estimated by seeking the grouping in which the experimental impedance is best approximated by the theoretical impedance. The possibility of simultaneously identifying these parameters can be discussed on the basis of a sensitivity study. This is performed by observing variations in the given function when subjected to a change in one of the parameters. Thus, analysis takes into account the range of variation and meets the conditions for de-correlating the quantities. In this way, the frequency range under study can be optimized as a function of the required objectives. It may even enable the model to be reduced as certain parameters would prove to have negligible influence on the observation range.

The impedance is a complex function of frequency. The sensitivity of the moduli and phases to various parameters is studied in parallel. The sensitivity functions S_{P_i} of both moduli and the phase of Z to the parameter p_i will be defined by the relation,

$$s_{Z,P_i}(f) = \frac{\Delta Z/Z}{\Delta p_i/p_i} \tag{11}$$

In this expression, Z represents alternatively the modulus or argument of the impedance. In order to make interpretation easier over a wide frequency range, the ratio of the relative variations in the parameter to the response function is calculated and expressed as a percentage of the tested function. Since $S_{z,p}$ is defined as a ratio of two non-dimensional functions, it follows that it is also non-dimensional. The calculation, numerically obtained, involves introducing nominal values of the various parameters. Such a constraint is not in contradiction with the aim of determining these parameters. The sensitivity functions are only used qualitatively. They highlight the predominance of given parameters in the behavior of the response function and their possible correlation. They also make it possible to choose optimum frequency ranges for identifying the main parameters. A rough estimate of the values is enough for this use of the sensitivity functions. In this case, the following values were chosen: $C_f = 800\,\mathrm{J \cdot m^{-2} \cdot K^{-1}}$ for the sensor capacity, $R_c = 4 \times 10^{-2}\,\mathrm{K \cdot m^2 \cdot W^{-1}}$ for the contact resistance, and $b = 2000\,\mathrm{J \cdot m^{-2} \cdot s^{-1/2} \cdot K^{-1}}$ for the thermal effusivity of the material.

Figure 3 shows a global view (for better appreciation) of the modulus sensitivity for each parameter. This example makes clear the choice of the frequency range between $\sim 10^{-3}$ Hz and some 10^{-2} Hz, because, in this range, the impedance is highly sensitive to the thermal effusivity of the material; it should also be noted that there is high sensitivity to resistance R_c which increases with frequency, and both sensitivity curves b and R_c are not proportional, indicating that the sensitivities are not correlated in the studied frequency range. The two parameters can be determined simultaneously. It should be noted that the sensitivity to the sensor capacity is much lower. Capacity values obtained by identification will be assigned a high level of uncertainty, because of a low sensitivity in the impedance estimation. The result could be fixed at a nominal value without hampering any identification of the others.

Assuming a lower frequency range, we could neglect the R_c parameter; however, in such a case, an experiment of sufficiently long time should be required that questions the uni-directionality of the exchanges.

Finally, to check if parameters b and R_c are correlated, the sensitivity to R_c is plotted as a function of sensitivity to b. If those parameters are correlated, the graph will be a line going through the origin.

Both sensitivities are non-dimensional and expressed in a percent form as in Fig. 4.

Hence, with regard to this study, parameters b and R_c can be simultaneously determined in the studied area, from the same test.

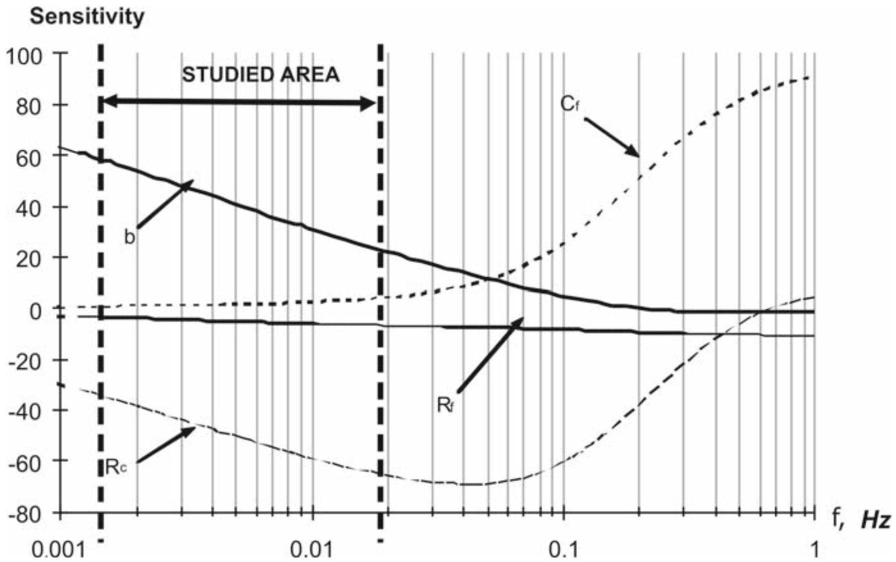

Fig. 3 Modulus sensitivity to parameters as a function of frequency

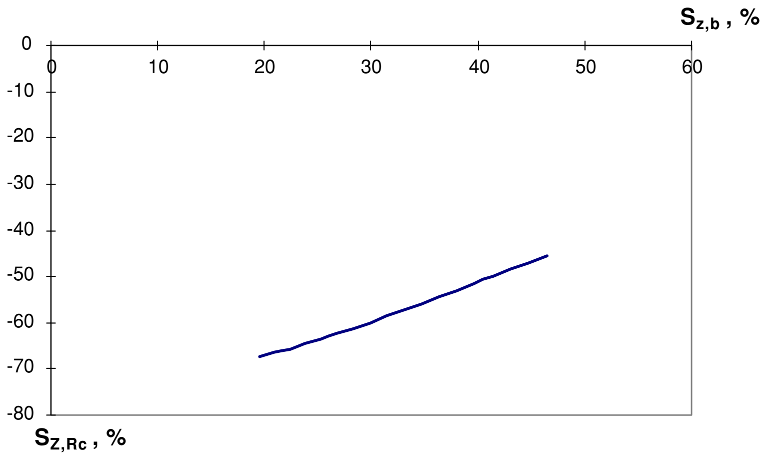

Fig. 4 Evolution of the sensitivity to the resistance R_c depending on the sensitivity to the thermal effusivity b

4 Model and Processing Validation Stage

4.1 Preliminary Characterization of the Sample

Characterization was carried out on a concrete sample. This was a typical case in which surface roughness produces high contact resistance values, up to $10^{-2}\text{K}\cdot\text{m}^2\cdot\text{W}^{-1}$. Although the material is naturally heterogeneous, the size of the sensor allows the heterogeneities to be integrated and we can consider it as homogeneous. A study of the heat transfer through the sample made it possible to determine a reference thermal conductivity value [9]. This study was carried out in the usual way with a conduction test bench. Analysis of a storage process in the same setup allowed us to measure the thermal capacity. On the basis of these preliminary measurements (determination of both thermal conductivity and specific heat) [10], it was possible to deduce a reference thermal effusivity value with the relation:

$$b = \sqrt{\lambda \rho c} \tag{12}$$

where λ is the thermal conductivity, and ρc is the volumetric specific heat.
Results are illustrated in Table 1.

4.2 Model Validation Stage

In order to estimate the experimental impedance, a model with the same derivative orders as the theoretical expression is chosen and coefficients α_i and β_j are determined from the measurements. The impedance was estimated by a harmonic study in order to validate the theoretical model. Determining the transfer function by a harmonic study requires the impedance to be plotted point by point after sufficiently long tests, but the result obtained can be used as a reference.

The validation procedure involves subjecting the sensor/material system to a sinusoidal flux density signal, i.e., one with a single frequency component. To this end, the generated power supply signal is shaped as a sine wave;

$$P(t) = p_0 \left(1 + \sin(2\pi f t + \varphi)\right) \tag{13}$$

The system is thermally driven by a heating resistance, controlled by a microcomputer.

The signal is generated with the command card (D.A.C card: digital-to-analog converter).

Convenient representative frequencies were chosen in the selected range of the studied area (Table 2).

Flux excitation is shaped as a continuous square pulse superimposed with a variable sinusoidal component. As the system is linear, the temperature response is equal to

Table 1 Results obtained on the concrete sample

Thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)	1.94
Volumetric heat capacity ($10^6\,J \cdot m^{-3} \cdot K^{-1}$)	1.6
Thermal effusivity ($10^3\,J \cdot m^{-2} \cdot s^{-1/2} \cdot K^{-1}$)	1.76

Table 2 Impedance moduli derived from the harmonic study at the various frequencies studied

Frequency (Hz)	0.005	0.00568	0.00666	0.00793	0.01	0.0125
Moduli ($K \cdot m^2 \cdot W^{-1}$)	0.0077	0.0076	0.007	0.0066	0.0062	0.0058

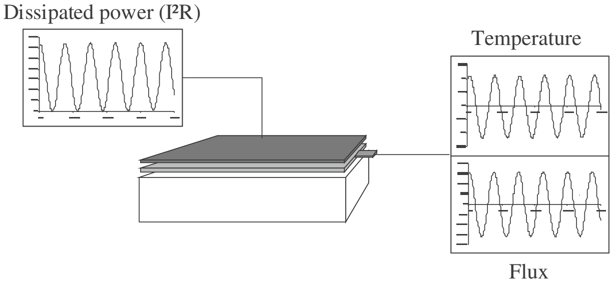

Fig. 5 Experimental procedure diagram

the sum of each frequency response. When the transient regime of response to the square pulse is assumed completed, the transient part of the temperature is sinusoidal with the same frequency as the stress (Fig. 5). The ratio between the amplitude of the harmonic component of the temperature to that of the flux yields the impedance Z modulus value for the given frequency of the signal and the phase shift yields the argument of Z.

For this frequency, the amplitude ratio gives

$$|Z\,(0.005)| = \left|\frac{\theta\,(0.005)}{\varphi\,(0.005)}\right| = 0.0077\ \mathrm{K}\cdot\mathrm{m}^2\cdot\mathrm{W}^{-1} \tag{14}$$

The obtained results for various frequencies are summarized in Table 2. The range of tested frequencies fits the spectral range targeted in this paper during characterization under pseudo-random test signals.

Thermophysical parameters of the theoretical model are identified by minimizing a deviation function between the experimental points and those derived from the theoretical curve considering solving the quadratic minimization problem (least-squares method). As the impedance is a non-linear function of the parameters, the approximation is obtained with an iterative algorithm. The simplex method [11] was used in this work. The initial values of the thermophysical parameters introduced into the algorithm were sufficiently close to the effective values of the system to avoid any problem of divergence or convergence towards a local minimum.

Figure 6 shows the points derived from the harmonic study and the optimized impedance curve. We can see good agreement between experimental and optimized curves. The theoretical curve is obtained with the following thermophysical parameter values: contact resistance = $0.0056\,\mathrm{K}\cdot\mathrm{m}^2\cdot\mathrm{W}^{-1}$, sensor capacity = $950\,\mathrm{J}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-1}$, and thermal effusivity = $1.72\times 10^3\ \mathrm{J}\cdot\mathrm{K}^{-1}\cdot\mathrm{m}^{-2}\cdot\mathrm{s}^{-1/2}$ (Table 3).

The thermal effusivity value is quite close to that obtained during the preliminary tests. Such good agreement regarding thermophysical property values and the close superposition of the experimental points and theoretical curve demonstrates the validation of the theoretical model.

The experimental impedance of this same system was then estimated with the aforementioned method, relying on non-integer models with parameters α_i and β_j. To avoid modifying any experimental condition, no work was done on the setup between both

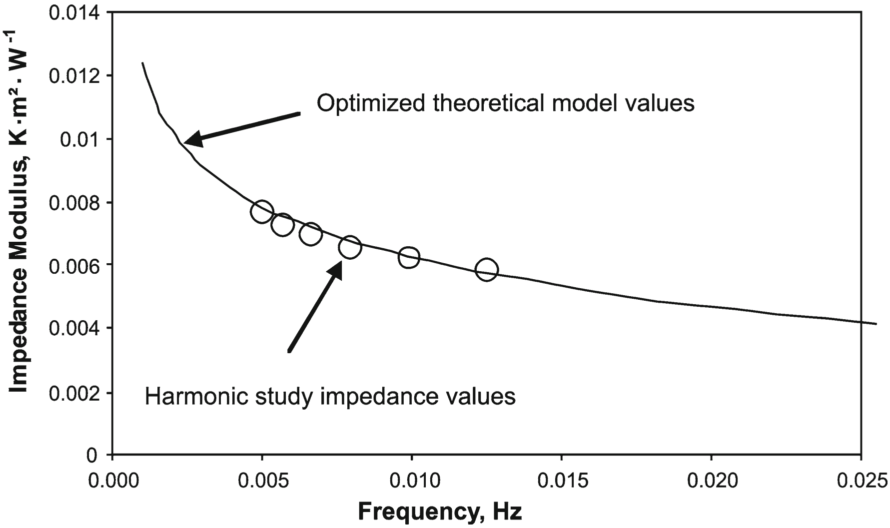

Fig. 6 Comparison of the impedance moduli and values found by the harmonic study

Table 3 Thermophysical parameters derived from the harmonic study

Contact resistance ($K \cdot m^2 \cdot W^{-1}$)	0.0056
Capacity of the sensor ($J \cdot m^{-2} \cdot K^{-1}$)	950
Thermal effusivity ($10^3\,J \cdot m^{-2} \cdot s^{-1/2} \cdot K^{-1}$)	1.72

stages of testing. Thus, contact conditions remained unchanged in order to guarantee a constant contact resistance value.

The resistance is supplied with an on/off signal so that the heating power is dissipated in the form of a pseudo-random binary signal (PRBS) (Fig. 7). PRBS [12] are signals easily generated by way of a series of high and low levels defined by deterministic laws; their statistical properties can be simulated as, and can be shown to be equivalent to, random processes [5]. Their spectrum is very rich as a frequency range, the upper limit depending on the rate of change of state and on the maximum number of bits included in the PRBS.

Figure 8 shows variations in flux density and surface temperature measured for a test carried out under pseudo-random stresses.

Entering the time-varying flux and temperature data into Eq. 6 enables the model parameters α_i and β_j to be estimated. The values determined from this test are given in Table 4.

The values were introduced into the impedance expression. The results are plotted in Fig. 9.

Figure 10 shows that the impedance values obtained by processing agree with the results of the harmonic study. Approximating the experimental impedance with the theoretical impedance yields parameter values that are very close to the previous ones, especially using the thermal effusivity value $b = 1.73 \times 10^3 J \cdot m^{-2} \cdot s^{-1/2} \cdot K^{-1}$.

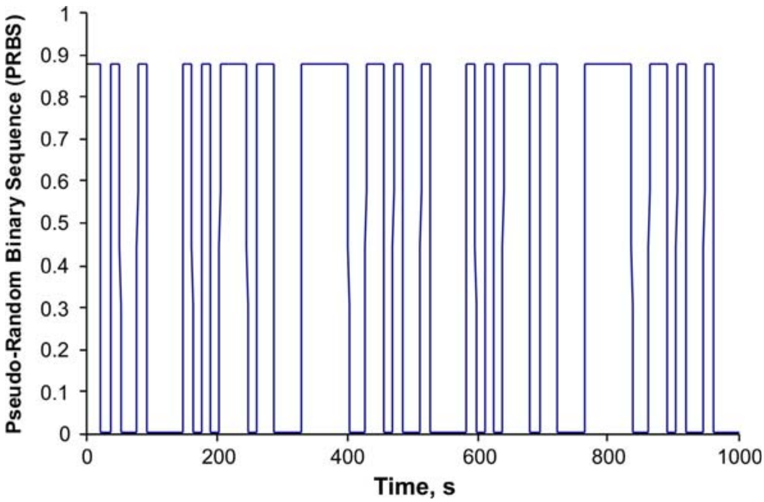

Fig. 7 Resistance supply voltage

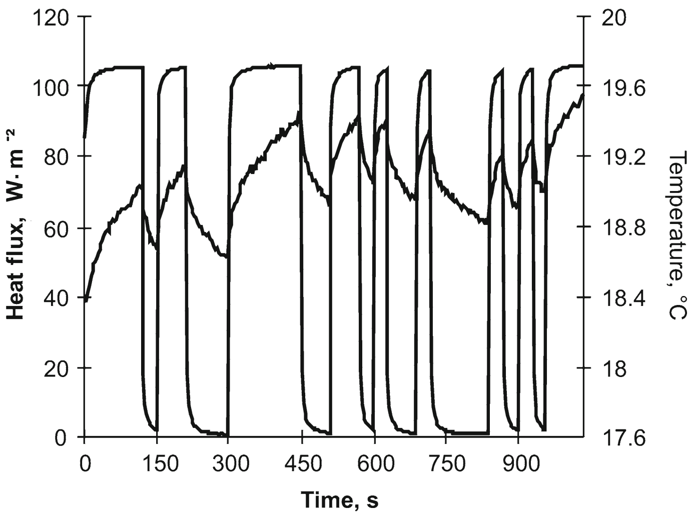

Fig. 8 Flux signals and temperature signals from a characterization test under random stresses

Table 4 Values of model parameters α_i and β_j estimated from Eq. 6 for this test

Parameters	α_1	α_2	α_3	β_1	β_2
Values	0.181234	0.118043	1	0.000097	0.001088

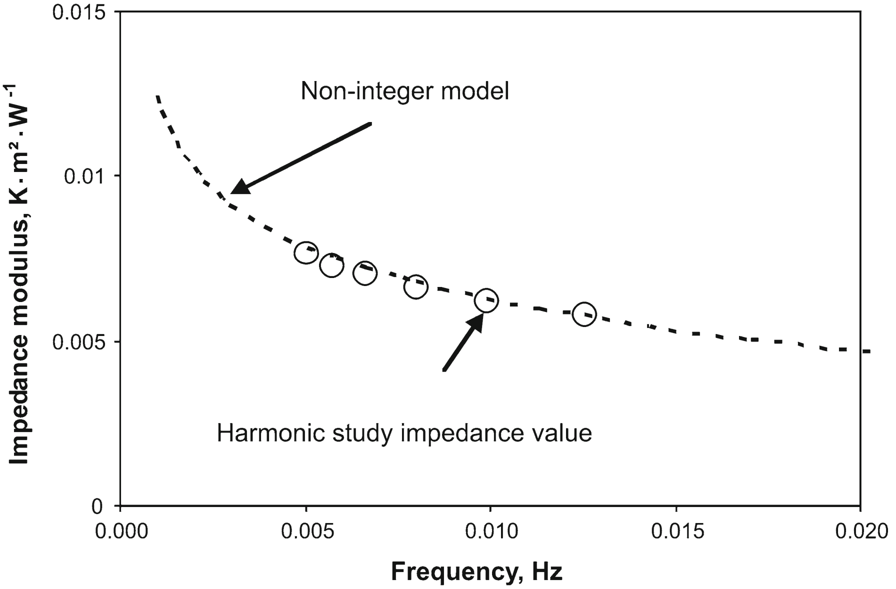

Fig. 9 Comparison of the impedance model based on the non-integer order model and values found with the harmonic study

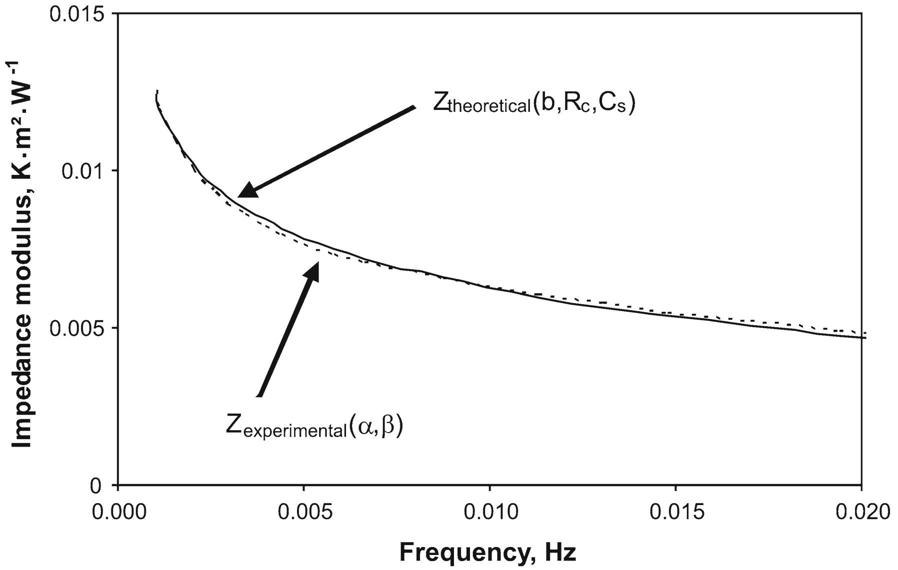

Fig. 10 Result based on optimizing the theoretical impedance (b,R_c,C_f) versus the experimental impedance (α_i,β_j)

5 Measurements of a Series of Tests

The procedure for determining parameters from a non-integer order model was applied to a series of 10 tests. Between such tests, the sensor was removed and then replaced on

Table 5 Application of the procedure to a series of ten tests for determining parameters from a non-integer order

Tests	1	2	3	4	5	6	7	8	9	10
b (10^3 J · m^{-2} · s$^{-1/2}$ · K^{-1})	1.73	1.82	1.81	1.83	1.78	1.82	1.71	1.79	1.81	1.86
R_C (10^{-3} K · m^2 · W^{-1})	5.7	4.2	4.8	5.3	5.7	5.5	2.9	6.1	3.3	4.2
C_f (10^2 J · m^{-2} · K^{-1})	9.7	9.6	9.5	9.8	9.5	9.5	9.3	9.6	9.7	9.7
$\bar{b}$ (10^3 J · m^{-2} · s$^{-1/2}$ · K^{-1})	1.8									
σ (10^3 J · m^{-2} · s$^{-1/2}$ · K^{-1})	0.046									
C_v (%)	3.5									

the surface of the material so as to modify the contact conditions, inducing significant changes of the thermal contact resistance.

Thermal effusivity values obtained in this way display significant reproducibility, whereas the resistance varies considerably. Even though the sensitivity study showed that it had little effect on the impedance, the thermal capacity of the sensor does not entail any significant variation.

Table 5 summarizes the results regarding thermal effusivity for a series of 10 tests.

6 Conclusion

This study demonstrates that a non-integer order model theory can be used to determine the frequency thermal impedance of a conduction system experimentally. A theoretical impedance model was defined on the basis of the quadrupole theory. Both the experimental contact resistance and capacity of the sensor were integrated in this model. The theoretical impedance exhibits non-integer orders of differentiation in the relation linking both the temperature and flux density over the input "access plane" of the system. The model was validated with a harmonic study. With pseudo-random heating signals, the model parameters were determined on the basis of flux and temperature measurements. Then direct access is enabled for the impedance in the frequency domain. An identification procedure allowed us to determine both the thermal effusivity of a material and the contact resistance. Several tests were carried out while changing the contact conditions for each test. This method, applied to a sample of concrete with a very high contact resistance, proved to be reliable.

Acknowledgments This work was supported by the "Conseil Régional du Nord-Pas de Calais" and takes place within the research program called "Ingénierie Urbaine".

References

1. D. Defer, E. Antczak, B. Duthoit, Meas. Sci. Technol. **12**, 1 (2001)
2. A. Oustaloup, *La dérivée non entière: théorie, synthèse et applications* (Hermès, Paris, 1995)
3. E. Delacre, D. Defer, E. Antczak, B. Duthoit, High Temps. - High Press. **33**, 337 (2001)
4. A. Short, W. Kinniburg, *Lightweight Concrete* (Galliard, Great Yormouth, UK, 1978), p. 113
5. D. Defer, E. Antczak, B. Duthoit, Meas. Sci. Technol. **9**, 496 (1998)
6. P. Hérin, P. Théry, Meas. Sci. Technol. **3**, 158 (1993)

7. A. Degiovanni, Int. J. Heat Mass Transfer **31**, 553 (1988)
8. J.L. Battaglia, L. Le Lay, J.C. Batsale, A. Oustaloup, O. Cois, Int. J. Therm. Sci. **39**, 374 (2000)
9. D. Leclercq, *Caractérisation des transferts thermiques par analyse de la réponse en fréquence* (Ph.D. Thesis, Lille, 1982)
10. L. Other-Duthoit, *Application des méthodes de la théorie des systèmes à la simulation de l'évolution des flux thermiques sur les faces d'entrée et de sortie d'une paroi multicouche* (Ph.D. Thesis, Lille, 1985)
11. J.C. Lagarias, J.A. Reeds, J.H. Wright, P.E. Wright, Soc. Indust. Appl. Math. **9,** 112 (1998)
12. R. Pintelon, J. Schoukens, *System Identification: A Frequency Domain Approach* (IEEE Press, Piscataway, New Jersey, 2001), ISBN 0-7803-6000-1